$$J_k(r, z, t) = \int_0^\infty T_k(q)\,dq, \quad T_k(q) = q^{-1}\cos(Q_k)\cos\left(qr - \frac{\pi}{4}\right)$$

We will assume the frequency passband of the recording apparatus to be constrained to the band $\omega_1 \leqslant \omega \leqslant \omega_2$. Let $\Delta\tau$ be the observation time which is sufficient to record these frequencies, where $\Delta\tau > \max\{2\pi/\omega_1, 2\pi/(\omega_2 - \omega_1)\}$. We shall also assume that $\Delta\tau \ll t$, where t is the time of wave packet propagation to the vibration-receiver. Then we can write

$$J_k(r, z, t) \approx \int_{q_1}^{q_2} T_k(q)\,dq, \quad q_i \sim \frac{\omega_i}{2\pi C_1}$$

instead of the improper integral in (4.4).

Inverting (4.1), we obtain formulas for the displacement in the far zone

$$u(r, z, t) = U(r, z, t) ** \sigma_{zz}(r, 0, t) \tag{4.5}$$
$$w(r, z, t) = W(r, z, t) ** \sigma_{zz}(r, 0, t)$$
$$A(r, t) ** B(r, t) = \frac{1}{\pi}\int_0^t\int_0^\pi\int_0^L A(\eta, \tau)\,B[(\eta^2 + r^2 - 2r\eta\cos\varphi)^{1/2},\ t - \tau]\,\eta\,d\eta\,d\varphi\,d\tau$$

The limit of integration $L(t)$ is determined by the boundaries of the domain within whose limits the stresses $\sigma_{zz}(r, 0, t)$ are non-zero.

Note that the relationships obtained are found for times when multiple wave diffraction by the crack edge is not taken into account. For later times (for a more accurate estimate of the energy travelling to the crack edges) the analysis of the problem is more complicated.

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FRACTURE OF BRITTLE BODIES WITH PLANE INTERNAL AND EDGE CRACKS *

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A brittle half-space with one or two symmetric plane edge cracks experiencing plane strain is considered. Compressive stresses act at infinity to cause superposition of opposite edges of the cracks (crack) and their mutual slip. Moreover, biaxial tension at infinity is considered for an unbounded brittle body with a plane crack. The body is under plane strain conditions and is stretched in two mutually orthogonal directions, one of which agrees with the direction of the crack. Limits of applicability are determined for the solutions of these problems by utilizing a well-known fracture criterion /1—4/, and a more general solution is given, resulting in a technical strength condition for the brittle bodies under consideration when there is no mutual displacement of the crack edges.

1. General remarks.

The generalization is made on the basis of the following principle /5/: all volumes included with a sphere of diameter

$$\Delta \approx \frac{4}{3}\frac{ET}{S_0^2} \tag{1.1}$$

are equally strong if the maximum (ε_1) and minimum (ε_3) relative elongations of the diameters

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of these spheres are, respectively, identical, and the relative changes in the volume (ε) inside them are identical. Here E is Young's modulus, T is the surface energy density, and S_0 is the resistance to cleavage (the stress at which brittle rupture of a uniaxially stretched rod occurs).

The quantities /5/

$$\sigma_k = 2G\left[\varepsilon_k + \nu\frac{\varepsilon}{1-2\nu}\right], \quad k = 1, 3 \tag{1.2}$$
$$\sigma_2 = 2G\left[(1+\nu)\frac{\varepsilon}{1-2\nu} - \varepsilon_1 - \varepsilon_3\right]$$

are called macrostresses (fundamental), where G is the shear modulus, and ν is Poisson's ratio.

It is considered that cleavage cracks are formed in a solid if the strength condition /5/

$$\sigma_1 + \frac{S_0}{S_*}(|\sigma_2| + |\sigma_3|) < S_0 \tag{1.3}$$

is violated where S_* is the absolute value of the stress for which brittle fracture of a uniaxially compressed rod occurs.

2. Biaxial loading of an unbounded body with a crack. Let us introduce a system of Cartesian coordinates x, y with origin at the centre of the crack and x axis coincident with the line of the crack. Tension is produced at infinity in the x, y axial directions by the stresses σ_x°, σ_y°, respectively.

The asymptotic form of the displacement field u_x, u_y and the volume strain ε has the following form in the case of plane strain /6/

$$2Gu_x = \frac{1}{2}\left(\frac{r}{2\pi}\right)^{1/2} K_I\left((2\varkappa-1)\cos\frac{\theta}{2} - \cos\frac{3\theta}{2}\right) - \frac{1}{4}(\varkappa+1)(\sigma_y^\circ - \sigma_x^\circ)(l + r\cos\theta) \tag{2.1}$$
$$2Gu_y = \frac{1}{2}\left(\frac{r}{2\pi}\right)^{1/2} K_I\left((2\varkappa+1)\sin\frac{\theta}{2} - \sin\frac{3\theta}{2}\right) - \frac{1}{4}(\varkappa-3)(\sigma_y^\circ - \sigma_x^\circ)\, r\sin\theta$$
$$2G\varepsilon = (\varkappa-1)\left[\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2} + \frac{1}{2}(\sigma_x^\circ - \sigma_y^\circ)\right], \quad \varkappa = 3 - 4\nu$$

where l is half the crack length $l \gg \Delta$, and r, θ are polar coordinates with origin at the apex $x = l$ of the crack.

The relative elongation ε_n of a sphere of diameter Δ in an arbitrary direction n in the x, y plane is evaluated from the formula /7/

$$\varepsilon_n = \frac{1}{\Delta}\left[(u_x(r_1,\theta_1) - u_x(r_2,\theta_2))\cos\psi + (u_y(r_1,\theta_1) - u_y(r_2,\theta_2))\sin\psi\right] \tag{2.2}$$

where r_0, θ_0 are the polar coordinates of the centre of the sphere, ψ is the angle made by the direction n with the x-axis, and r_1, θ_1 and r_2, θ_2 are the polar coordinates of the intersections of the sphere with a line passing through the centre of the sphere in the direction n

$$r_{1,2} = [r_0^2 \pm \Delta r_0\cos(\theta_0 - \psi) + (\Delta/2)^2]^{1/2}$$
$$\operatorname{tg}\theta_{1,2} = (2r_0\sin\theta_0 \pm \Delta\sin\psi)(2r_0\cos\theta_0 \pm \Delta\cos\psi)^{-1}$$

The macrostrains ε_1, ε_3 are the maximum and minimum values, respectively, of the relative elongation as the angle ψ varies in the range $0 \leqslant \psi < \pi$.

The relative change (ε) in the volume of any sphere equals the volume expansion at the centre of this sphere. Hence, it is not difficult to determine the macrostress using the asymptotic formulas (2.1).

Computations by the strength condition (1.3) and (2.1)-(2.2) showed that the maximum macrostresses near the tip of the crack are reached in directions making angles $\pm\beta$ with the crack direction; these macrostresses are practically constant as β changes by $\pm 15°$.

Under uniaxial tension in a direction orthogonal to the crack, the angle β is approximately $77°$, and the maximum macrostress exceeds the corresponding macrostress when the crack is extended by approximately 25% /5/ (depending on Poisson's ratio). An analogous result is presented in /8/ for idealized stresses (in conformity with the standard concept of stresses at a point), where the angle β equals $60°$.

By using the strength condition (1.3), we obtain the crack-formation load under biaxial tension. A graph of this load is shown in Fig.1, where σ_t is the crack-formation load under uniaxial tension orthogonal to the crack. It is assumed in the computations that $\nu = 0.25$, $S_0/S_* = 0$, while the approximate formula $\sigma_t \approx (\Delta/2l)^{1/2}$ holds for σ_t.

It follows from the graph that biaxiality of the loading substantially influences the fracture of a body only if the tensile stress σ_x° exceeds the stress orthogonal to the crack (σ_y°). For $\sigma_x^\circ \lesssim \sigma_y^\circ$ the crack-formation load is close to a constant and agrees with the

Griffiths fracture load when (1.1) is used for the structural parameter.

When the crack-formation load is reached, approximately identical macrostresses hold at the end of the crack for different microvolumes in a certain domain. Consequently, the directions of macrocrack propagation are responsive to structural inhomogeneities. There can hence be several directions of macrocrack propagation (crack bifurcation). Any direction for which the crack length has a real value is possible (in a certain range) from the viewpoint mentioned. Those directions for which the length of the microcracks that originate is greatest will obviously be most probable. The length of the microcracks is here governed by the fact that the macrostresses at its tip satisfy condition (1.3) at which the inequality sign becomes an equality.

Under uniaxial tension in a direction orthogonal to the crack, crack development in its initial direction will be most probable despite the fact that the most stresses microvolumes in the initial crack-formation phase are located at an angle to the crack continuation. The mainline crack occurs from within the body in the plane of symmetry after the redistribution of the stress caused by fracture of the most stressed microvolumes (the appearance of microcracks that are not being propagated). If the fracture load σ_t° is here known from experiment, then the magnitude of the structural parameter $\Delta \approx 2l\,(\sigma_t^\circ/S_0)^2$ is determined approximately from the last formula /5/. But, nevertheless, we use (1.1) for comparison with solutions which are obtained by using the well-known fracture criterion /1—4/. This formula follows from the preceding one if the experimental quantity σ_t° is replaced by the Griffith fracture load.

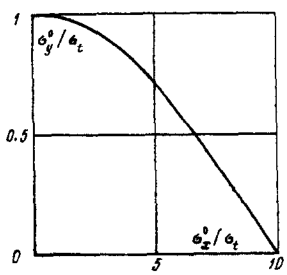

Fig.1

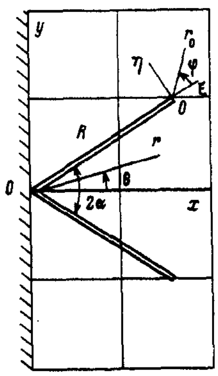

Fig.2

The disadvantage of the well-known method of computation is that the crack-formation load is independent of the stress σ_x° and when the inequality $\sigma_y^\circ < \sigma_t^\circ$ is satisfied the strength of the body with the crack will be unlimited. Utilization of the macrostress concept and the strength condition (1.3) eliminates these disadvantages and enables the influence of biaxiality of the loading on the fracture of brittle bodies with a crack to be estimated (Fig.1).

3. **Brittle half-plane with two symmetric edge cracks.** In this case a quadrant with one crack (Fig. 2) can be considered because of symmetry. The state of stress of a body will be homogeneous if $K^\circ_\alpha \leqslant 0$, where

$$K^\circ_\alpha{}' = \sigma_0 K_\alpha - \tau_c$$

$$K_\alpha = \cos\alpha\,(\sin\alpha - \eta_0 \cos\alpha)$$

2α is the angle between the crack directions, η_0 is the coefficient of friction, τ_c is the adhesion coefficient, and σ_0 is the absolute value of the compressive stress at infinity. When this condition is violated, slipping of the crack edges occurs. The boundary conditions here take the form

$$\theta = 0: u_\theta = \tau_{r\theta} = 0,\ r \geqslant 0 \tag{3.1}$$

$$\theta = \frac{\pi}{2}: \sigma_\theta = \tau_{r\theta} = 0,\quad r \geqslant 0 \tag{3.2}$$

$$\theta = \alpha: [\sigma_\theta] = [\tau_{r\theta}] = [u_\theta] = 0,\ r \geqslant 0 \tag{3.3}$$

$$\theta = \alpha: \tau_{r\theta} = \eta_0\sigma_\theta + K_\alpha^\circ,\ 0 \leqslant r < 1 \tag{3.4}$$

$$\theta = \alpha: [\sigma_r] = 0,\ r > 1 \tag{3.5}$$

Here r is the polar radius-vector referred to the crack length $R; R \gg \Delta$ and θ is the polar angle. The components σ_{ik}, u_i are the difference between the total stresses and displacements and the corresponding quantities in a body without cracks. The square brackets denote a jump in the quantity enclosed in the brackets when the line of the crack is passed through

$$[\sigma_{ik}] = \sigma_{ik}(r, \alpha + 0) - \sigma_{ik}(r, \alpha - 0)$$
$$[u_i] = u_i(r, \alpha + 0) - u_i(r, \alpha - 0)$$

The condition at infinity

$$\sigma_{ik} = o(r^{-1}), \quad r \to \infty, \ 0 \leqslant \theta \leqslant \pi/2 \tag{3.6}$$

must be added to the relationships (3.1)-(3.5).

The general solution $\sigma_{ik}^*, u_r^*, u_\theta^*$ of the homogeneous plane problem of the theory of elasticity in Mellin transforms for the stresses σ_{ik} and the displacements u_r, u_θ (referred to the characteristic dimension R) has the form

$$\begin{aligned}
2Gu_r^* &= (p + \varkappa)(Ac_+ + Bs_+) + (p - 1)(Cc_- + Ds_-) \qquad (3.7)\\
2Gu_\theta^* &= (p - \varkappa)(As_+ - Bc_+) + (p - 1)(Cs_- - Dc_-)\\
\sigma_r^* &= -p\,[(p + 3)(Ac_+ + Bs_+) + (p - 1)(Cc_- + Ds_-)]\\
\sigma_\theta^* &= p(p - 1)[Ac_+ + Bs_+ + Cc_- + Ds_+]\\
\tau_{r\theta} &= -p[(p + 1)(As_+ - Bc_+) + (p - 1)(Cs_- - Dc_-)]\\
c_\pm &= \cos(p \pm 1)\theta, \ s_\pm = \sin(p \pm 1)\theta
\end{aligned}$$

where $\varkappa = 3 - 4\nu$ for plane strain, and $\varkappa = (3 - \nu)/(1 + \nu)$ for the plane-parellel state, and A, B, C, D are arbitrary functions of the complex parameter p determined from the boundary conditions. We shall consider the coefficients A_1, B_1, C_1, D_1 to correspond to a wedge with aperture $0 \leqslant \theta \leqslant \alpha$; and A_2, B_2, C_2, D_2 to correspond to the wedge $\alpha \leqslant \theta \leqslant \pi/2$.

The boundary conditions (3.1),(3.3),(3.5) are satisfied if we set

$$\begin{aligned}
B_1(p) &= D_1(p) = 0\\
A_2(p) &= A_1(p) - \cos(p + 1)\alpha\,\Phi^-(p)/(4p)\\
B_2(p) &= -\sin(p + 1)\alpha\Phi^-(p)/(4p)\\
C_2(p) &= C_1(p) + \cos(p - 1)\alpha\Phi^-(p)/(4p)\\
D_2(p) &= \sin(p - 1)\alpha\Phi^-(p)/(4p)\\
\Phi^-(p) &= \int_0^1 [\sigma_r(r, \alpha)]\,r^p\,dr
\end{aligned}$$

Inserting these expressions into the boundary conditions (3.2) and solving the equations obtained, we will have

$$A_1(p) = S_1(p)\Phi^-(p)/(4p\sin p\pi)$$
$$C_1(p) = S_2(p)\Phi^-(p)/(4p\sin p\pi)$$

$$S_1(p) = (p - 1)\sin(p - 1)\alpha - p\sin((p + 1)\alpha - \pi) + \sin(p\pi - (p + 1)\alpha)$$
$$S_2(p) = (p + 1)\sin(p + 1)\alpha - p\sin((p - 1)\alpha + \pi) - \sin(p\pi - (p - 1)\alpha)$$

The transforms of the normal and shear stresses on the line of the crack become

$$\sigma_\theta^* = (p - 1)[S_1(p)\cos(p + 1)\alpha + S_2(p)\cos(p - 1)\alpha]\,\Phi^-(p)/q \tag{3.8}$$
$$\tau_{r\theta}^* = -[(p + 1)S_1(p)\sin(p + 1)\alpha + (p - 1)S_2(p) \times \sin(p - 1)\alpha]\Phi^-(p)/q$$
$$q = 4\sin p\pi$$

Substitution of the transforms obtained into the Mellin-transformed friction condition (3.4) results in a functional Wiener-Hopf equation,

$$\begin{aligned}
&-\frac{1}{4}\,\mathrm{tg}\,p\frac{\pi}{2}\,G_0(p)\,\Phi^-(p) = \Phi^+(p) + \frac{K_\alpha^\circ}{p + 1} \qquad (3.9)\\
&G_0(p) = \frac{G_1(p) + 2\eta_0(p - 1)\,G_2(p)}{2\sin^2 p\frac{\pi}{2}}\\
&G_1(p) = (\sin 2p\alpha + p\sin 2\alpha)\left(\sin 2p\left(\frac{\pi}{2} - \alpha\right) + p\sin 2\alpha\right) +\\
&\qquad 2(\cos 2p\alpha - \cos 2\alpha)\left(\sin^2 p\left(\frac{\pi}{2} - \alpha\right) - p^2\cos^2\alpha\right)\\
&G_2(p) = p\sin 2p\alpha\cos^2\alpha + \sin 2\alpha\sin^2 p\left(\frac{\pi}{2} - \alpha\right)\\
&\Phi^+(p) = \int_1^\infty (\tau_{r\theta}(r, \alpha) - \eta_0\sigma_\theta(r, \alpha))\,r^p\,dr
\end{aligned}$$

When there is no friction, the equation obtained in /9/ in connection with the initial development of the slip line from the free boundary of the body follows from (3.9).

The transforms desired $\Phi^\pm(p)$ are analytic in the half-planes $\operatorname{Re}(p) < 1$ and $\operatorname{Re}(p) > -1$ respectively. As $p \to \infty$, the following asymptotic formulas /10/ hold for these transforms

$$\Phi^-(p) = -2\sqrt{2}\,K_{II}/\sqrt{Rp}, \quad \Phi^+(p) = K_{II}/\sqrt{-2Rp}, \quad p \to \infty \tag{3.10}$$

where K_{II} is the stress intensity factor at the tip of the crack.

The coefficient $G_0(p)$ of (3.9) has neither poles nor zeroes along the boundary line $L = \{p = -1 + it, |t| < \infty\}$ of the strip of analyticity of the transforms $\Phi^\pm(p)$ if $\alpha > \rho_0$, where ρ_0 is the angle of internal friction $\eta_0 = \operatorname{tg}\rho_0$. Because of the condition $\alpha < \pi/2$ this coefficient tends to one as $p \to \infty$ along the line L. For $\alpha = \rho_0$ the function $G_0(p)$ has a first-order zero at $p = -1$

$$G_0(-1) = 4\sin 2\alpha K_\alpha$$

Let D^+ and D^- denote domains lying to the left and right of the line L, respectively. Then for values of $\rho_0 < \alpha < \pi/2$, the coefficient in (3.9) can be factorized thus

$$G_0(p) = \frac{G^+(p)}{G^-(p)} \tag{3.11}$$

$$\exp\frac{1}{2\pi i}\int\limits_L \frac{\ln G_0(s)\,ds}{s-p} = \begin{cases} G^+(p) & p \in D^+ \\ G^-(p) & p \in D^- \end{cases}$$

The functions $G^\pm(p)$ are analytic and have no zeroes in the domains $D^\pm$, hence $G^\pm(p) \to 1$ as $p \to \infty$.

To solve the inhomogeneous conjugate problem (3.9) on the line, we use the well-known representation /10, 11/

$$p\operatorname{ctg} p\frac{\pi}{2} = 2K^+(p)K^-(p) \tag{3.12}$$

$$K^\pm(p) = \Gamma\left(1 \mp \frac{p}{2}\right)\Big/\Gamma\left(\frac{1 \mp p}{2}\right)$$

By using the factorization (3.11) and the representation (3.12), Eq.(3.9) on the line L can be written as follows:

$$\frac{p\Phi^-(p)}{8K^-(p)G^-(p)} + \frac{K^+(-1)K_\alpha^\circ}{(p+1)G^+(-1)} = -\frac{K^+(p)\Phi^+(p)}{G^+(p)} - \frac{K_\alpha^\circ}{p+1}\left[\frac{K^+(p)}{G^+(p)} - \frac{K^+(-1)}{G^+(-1)}\right]$$

The left side of this equation is analytic in D^- and the right side in D^+. On the basis of the principle of continuous extension they equal the same entire function. Using the asymptotic form (3.10) and the well-known relationships

$$K^\pm(p) = \sqrt{\mp p/2} + O(1), \; p \to \infty$$

we obtain the formula

$$p\Phi^-(p) = -4K^-(p)G^-(p)\left[\frac{K_{II}}{\sqrt{R}} + \frac{\sqrt{\pi}K_\alpha^\circ}{(p+1)G^+(-1)}\right]$$

The absence of a pole in the transform $\Phi^-(p)$ at the point $p = 0$ determines the stress intensity factor and the desired solution

$$K_{II} = -\sqrt{\pi R}K_\alpha^\circ/G^+(-1) \tag{3.13}$$

$$\Phi^-(p) = 4\sqrt{\pi}K_\alpha^\circ K^-(p)G^-(p)/((p+1)G^+(-1))$$

Passing to the limit in $G^+(p)$ as $p \to -1$ and using the Sokhotskii-Plemelj formula /12/, we obtain

$$G^+(-1) = \sqrt{G_0(-1)}\exp\frac{1}{\pi}\int\limits_0^\infty \operatorname{Arg} G_0(-1+it)\frac{dt}{t} \tag{3.14}$$

Introducing a new rectangular coordinate system ξ, η with origin at the tip of the crack (Fig.1), and setting $r_0 \ll R$, we arrive at the asymptotic expressions

$$2Gu_\xi = \frac{1}{2}K_{II}\left(\frac{r_0}{2\pi}\right)^{1/2}\left((2\varkappa+3)\sin\frac{\varphi}{2} + \sin\frac{3\varphi}{2}\right) - \sigma_0\left[(\sin^2\alpha - \nu)\cos\varphi + \frac{1}{2}\sin 2\alpha\sin\varphi\right]r_0 \tag{3.15}$$

$$2Gu_\eta = -\frac{1}{2}K_{II}\left(\frac{r_0}{2\pi}\right)^{1/2}\left((2\varkappa-3)\cos\frac{\varphi}{2} + \cos\frac{3\varphi}{2}\right) - \sigma_0\left[\frac{1}{2}\sin 2\alpha\cos\varphi + (\cos^2\alpha - \nu)\sin\varphi\right]r_0$$

where r_0, φ are polar coordinates with origin at the crack apex, u_ξ, u_η are components of the total displacement vector in the directions of the coordinate axes ξ, η, respectively.

The formula for the total volume strain ε_s is found in a well-known manner

$$2G\varepsilon_s = -(\varkappa - 1)\left[\frac{K_{II}}{\sqrt{2\pi r_0}}\sin\frac{\varphi}{2} + \frac{1}{2}\sigma_0\right] \tag{3.16}$$

Using the strength condition (1.3), we obtain the crack-formation load. The dependence of this load on the angle α in the absence of adhesion is shown by the solid line in Fig.3, where σ_c is the minimum crack-formation load. It is assumed in the computations that $\nu = 0.25$, $\rho_0 = 10°$, $S_0/S_* = 0.1$. and the value of σ_c is determined from the approximate formula

$$\sigma_c \approx 1.7 S_0 \sqrt{\Delta/R} \tag{3.17}$$

On reaching the crack-formation load, here exactly as in Sec. 2 approximately identical macrostresses occur at the tip of the crack in a certain domain. Consequently, the direction of the originating microcracks will be considered to be random, and its length will be determined by the method indicated for the tension.

That direction for which the length mentioned is greatest is considered to be the most probable.

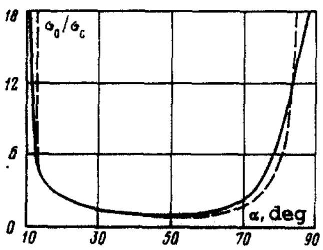

Fig.3

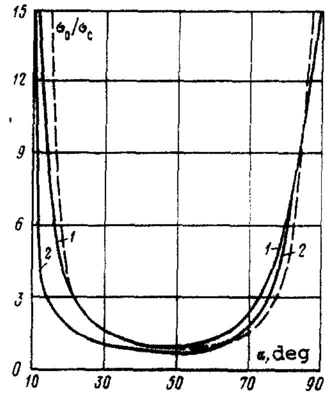

Fig.4

Comparison of the crack-formation load constructed by means of criterion (1.3) with the load determined by well-known methods /1–4/ is represented in Fig.3 by the dashed line. The loads obtained are close everywhere except the direct one $\alpha = \rho_0$ and $\alpha = \pi/2$. For these angles the stress intensity factor vaaishes and the criterion of maximum tensile stress /1–4/ results in unlimited strength.

4. **Brittle half-plane with an edge crack.** In this case the boundary conditions (3.2)-(3.5) remain unchanged, but the symmetry condition (3.1) is replaced by the condition of no stress on the lower part of the half-plane boundary

$$\theta = -\pi/2 : \sigma_\theta = \tau_{r\theta} = 0, \ r \geqslant 0 \tag{4.1}$$

Application of the Mellin transform to the boundary condition (3.2),(4.1),(3.3), and (3.5) determines the transform coefficients (3.7) in terms of one unknown function $\Phi^-(p)$

$$\begin{aligned}
A_1(p) &= (p \sin p\alpha \cos \alpha + c^-\delta)\Phi^-(p)/(pq)\\
B_1(p) &= (p \cos p\alpha \cos \alpha + s^-\delta)\Phi^-(p)/(pq)\\
C_1(p) &= (p \sin p\alpha \cos \alpha - c^+\delta)\Phi^-(p)/(pq)\\
D_1(p) &= (p \cos p\alpha \cos \alpha + s^+\delta)\Phi^-(p)/(pq)\\
A_2(p) &= A_1(p) - \cos(p+1)\alpha\, \Phi^-(p)/(4p)\\
B_2(p) &= B_1(p) - \sin(p+1)\alpha\, \Phi^-(p)/(4p)\\
C_2(p) &= C_1(p) + \cos(p-1)\alpha\, \Phi^-(p)/(4p)\\
D_2(p) &= D_1(p) + \sin(p-1)\alpha\, \Phi^-(p)/(4p)
\end{aligned}$$

$$c^\pm = \cos\left(p\frac{\pi}{2} \pm \alpha\right), \quad s^\pm = \sin\left(p\frac{\pi}{2} \pm \alpha\right), \quad \delta = \sin\left(p\left(\frac{\pi}{2} - \alpha\right)\right)$$

Substitution of these expressions in the Mellin-transformed friction condition (3.4) results in the functional Wiener-Hopf equation (3.9), where now

$$G_0(p) = -G_*(p) \Big/ \left(2\sin^2 p\frac{\pi}{2}\right) \tag{4.2}$$

$$C_*(p) = \cos p\pi - 2p\cos\alpha\,[(p-1)\eta_0 \cos\alpha + \sin\alpha] \times \sin 2p\alpha + (2p^2\cos^2\alpha - 1)\cos 2p\alpha$$

This equation is obtained and solved in /10/. The solution presented below differs from /10/ in the selection of the connecting line (L). For such a choice the coefficient (4.2) has neither poles nor zeroes along L if $\alpha > \rho_0$, and tends to one as $p \to \infty$. For $\alpha = \rho_0$ the function (4.2) has a first-order zero at $p = -1$

$$G_0(-1) = 2 \sin 2\alpha K_\alpha$$

The solution of (3.9) with the coefficient (4.2) and the appropriate quantity K_{II} have the form (3.13). Here $G^+(-1)$ is defined, as before, by (3.14) in which $G_0(p)$ has the form (4.2).

A computation by the strength condition (1.3) and (2.2),(3.15), and (3.16) shows that here, as in Secs.2 and 3, approximately identical macrostresses at the tip of the crack occur in a certain domain. Consequently, the considerations regarding the length and direction of the originating microcracks, discussed in the preceding sections, hold here.

The dependence of the crack-formation load on the angle α when there is no adhesion between the crack edges is shown in Fig.4 (curve *1*). As before, σ_c is understood to be the minimum crack-formation load when a body with one crack is compressed. The following approximate formula holds for it

$$\sigma_c \approx 2.24\, S_0 \sqrt{\Delta/R} \tag{4.3}$$

The computation is performed here for the same values of the parameters $\nu, \rho_0, S_0/S_*$ as in the previous section. The crack-formation load constructed by the well-known method /1–4/ is shown in Fig.4 by the dashed line. Exactly as in Sec.3, the results are close everywhere, except along directions close to the angle of internal friction and the boundary of the half-plane where the well-known method results in unlimited crack-formation loads.

The comparison of the crack-formation load when there is one crack (curve *1*) with the corresponding load when there are two symmetric cracks (curve *2*) is presented in Fig.4. The crack-formation loads in both cases are referred to the value of σ_c, defined by (4.3). It is seen that the presence of the second crack substantially reduces the crack-formation load in the range $\rho_0 \leqslant \alpha \leqslant 35°$. For angles close to $\pi/2$ the difference between the loads is erased, the latter grow rapidly as α grows, and approach the crack-formation load in a body without cracks.

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